



schools online curriculum content initiative

# **THE LE@RNING FEDERATION**

## **MATHEMATICS AND NUMERACY**

### **Phase 3**

### **Project Scoping Brief**

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# 1. Introduction

The Le@rning Federation (TLF) is an initiative of the Australian government, the governments of the Australian States and Territories and the New Zealand government. Its primary purpose is to develop online digital learning resources (curriculum content) in six agreed curriculum priority areas, one of which is Mathematics and Numeracy.

All online content procured and developed by The Le@rning Federation:

- aims to engage students in interactive learning experiences across a range of learning contexts
- comprises specific online learning opportunities that are connected to intended learning outcomes and that may be linked to individual State or Territory curriculums
- focuses on concept development and the transfer of skills and understandings to and from real-world domains while integrating with offline learning opportunities and making connections within and across areas of learning
- focuses on digital learning resources that can be used in conjunction with other resources in the classroom, for example, as part of a learning sequence that also incorporates classroom discussion, reading a text and writing responses
- encourages students to learn differently and more effectively
- provides a range of learning experiences not otherwise immediately or usually available to students in regular classroom contexts
- caters for individual learners or small groups and for various learning environments, for example, distance education settings
- provides opportunities for students to engage in meaningful interaction and communication
- challenges and encourages students to question, investigate and solve problems
- recognises that students have different needs, learn in many different ways and have preferences in learning modes
- aims to develop habits of mind for lifelong learning
- utilises new media and technologies for data input, interpretation, analysis and synthesis
- aims to enhance students' literacy and numeracy development, including specific critical literacy and numeracy skills necessary for effective interactions with ICTs, the Internet and multimedia
- provides teachers with opportunities to incorporate digital content within their learning and teaching program
- allows and encourages teachers and learners to use the mathematical and intellectual property incumbent in the content in a manner that complies with copyright law
- is useable within the agreed range of software and hardware platforms
- is packaged and presented in formats that can be readily contextualised by teachers and students within their teaching and learning environments.

These characteristics will continue to influence the development of learning objects in Phase 3.

In the first two phases of the initiative (Projects 1 to 5) Mathematics and Numeracy resources were developed for students across Years P to 9. Those five projects, now completed or nearing completion, are:

1. Counting and quantifying
2. Representing and visualising
3. Variation, transformation and change
4. Uncertainty and predictability
5. [No specific theme] Development of learning objects to complement and supplement those produced in Projects 1 to 4.

In Phase 3 (the Project), Mathematics and Numeracy digital resources will be procured and developed for students across Years 3 to 12. The audiences for the Mathematics and Numeracy online content being developed or acquired during 2006–09 are students in Years 3 to 12 who will use the content, and their teachers who will include the content in individual, group or whole-class teaching–learning programs.

This Project Scoping Brief (PSB) describes the purpose, objectives, content priorities and other requirements for the online curriculum content to be developed for Mathematics and Numeracy in Phase 3 of the initiative (the Project).

The Project Scoping Brief (PSB) is the result of the Mathematics and Numeracy Curriculum Area Reference Group (CARG) considering and discussing a draft prepared for its meeting on 27 November 2006, and subsequent development in the light of those discussions.

The audiences for this brief are the Mathematics and Numeracy CARG, The Le@rning Federation’s stakeholders, the Teachers Focus Group (TFG) for the Project, The Le@rning Federation staff and the Content design team.

## 2. Phase 3 Project in Mathematics and Numeracy

### 2.1 Purpose

The purpose of Phase 3 of the MCEETYA Online Curriculum Content Agreement 2006–09 is to continue the procurement of high-quality globally recognised online content for Australian and New Zealand schools.

The broad purpose of the Phase 3 Mathematics and Numeracy Project is to procure and produce high-quality, innovative online content that enhances the teaching and learning of Mathematics and the development of numeracy across Years 3 to 12.

### 2.2 Background

The identification of curriculum areas for content procurement and development for Phase 3 projects in all curriculum priority areas was provided by senior curriculum officers as an outcome of a Curriculum Forum held in August 2006. Further guidance was derived from analysis of national and international assessment data and from the results of the longitudinal research into the impact of digital content on student achievement and engagement.

In the case of Mathematics and Numeracy, in order to provide resources for topics in which students generally underperform, the objectives and scope of the Project have been further influenced by analysis of the performance of Australian and New Zealand students in recent international assessment studies – Trends in International Mathematics and Science (TIMSS 2003) and OECD Program for International Student Assessment (PISA 2003) – as well as the MCEETYA national numeracy benchmarks (2004), the New Zealand NEMP report (2005) and the New Zealand Numeracy Development Project research and evaluation findings compendia (2005 and 2006).

### 2.3 Objectives

The Mathematics and Numeracy Project for Phase 3 should procure or develop resources that:

- assist students to develop their problem-solving skills and strategies and enhance their performance in open-ended investigations
- develop and enhance students' command of mathematical and numerical processes, skills and conventions in ways that are epistemically sound
- acknowledge and take advantage of students' interest in and engagement with simulations and games
- target areas of student underperformance as identified in national and international student assessment reports
- are aligned with 'Statements of learning for Mathematics' (2006)
- focus on complex mathematical concepts.

As well, selected contexts and applications should be relevant to students' real and imagined (for example, in game simulations) worlds. Such real-world contexts will involve the use of mathematics to meet the personal demands of life at school (including those across all learning areas), at home, in paid work and in participation in community life. The numeracy demands of other learning areas and the situations and contexts in which these demands arise should be identified and used to inform the development of learning objects.

## 2.4 Content Priorities

The Project comprises the design and development of high-quality online digital resources in the form of learning objects for use at Years 3 to 12. The learning objects are easily disaggregated into numerous stand-alone learning objects.

Content priorities for the Project are organised as two (overlapping) parts, one part being concerned with digital content for Years 3 to 8/9 (the middle years) and the other with the content for Years 9 to 12 (the senior years).

Analysis of data on student underperformance in TIMSS, PISA and national testing, combined with a review of content covered by the Mathematics and Numeracy learning objects developed during the first two phases of the initiative (Projects 1 to 5), revealed concept and content areas deserving of further attention. While some of these areas of underperformance are addressed in the learning objects already developed, there is opportunity and need to develop further learning objects in these areas so as to provide greater continuity across Years 3 to 9 and beyond them.

The approach of identifying content areas in which students have underperformed in national and international tests to define some of the content for the Project has generated a substantial list of mathematical skills and concepts (see the three tables in 'The Middle Years (Years 3 to 8/9)', below). Development teams should be aware of the associated risk of learning objects acting to compartmentalise and 'atomise' mathematics learning rather than to present the big ideas and contexts that encourage students to engage with Mathematics. A balance of the two is needed.

### **Requirements of Indigenous students and students in remote areas**

Indigenous students irrespective of location, along with non-Indigenous students in remote areas, consistently underperform other students in all national and international tests. Further research is needed to identify whether or not this is the result of, for example, weaknesses in the provision of learning resources or failure to accommodate preferred learning styles. As part of that research effort, the Mathematics and Numeracy CARG recommends that some existing learning objects that align with the content selection for the new Project be included in trials to be conducted as part of TLF's separate Indigenous Research Project. Analysis of the results, along with further expert advice, could guide the production of new learning objects and the adaptation of existing learning objects. Such guidance could be added to a later version of this PSB.

## **The Middle Years (Years 3 to 8/9)**

*Priority: Areas of student underperformance*

*Priority: Epistemic approach to processes, skills and conventions*

The topic areas of 'decimals and fractions' and 'knowing facts and procedures' were identified by national and international testing programs as areas of weakness, particularly for students in Years 4 and 5. At Years 7 to 8/9 algorithms (the four basic operations with whole numbers, fractions and decimals) and algebraic processes were significant areas of student underperformance.

Further analysis of the data from the tests, along with consideration of gaps in the catalogue of existing learning objects revealed the content areas that are important for digital resource development within the Project (see table below).

A constant theme of the TIMSS and PISA reports on the performance of Australian and New Zealand students in the international testing programs was underperformance in solving basic problems presented in words. This underperformance cannot be attributed solely to students' reading skills or to the calculations involved in basic worded problems. The underlying

deficiency for a large proportion of students is their lack of understanding of the concepts of addition, subtraction, multiplication and division. Without this understanding it is difficult to interpret a problem and then to choose the appropriate operation(s) to perform for solving the problem.

Years 3 to 6/7	Years 7 to 8/9
<p><i>Students learn and consolidate facts, procedures and conventions through scaffolded tasks and meaningful practice in the areas of:</i></p> <p style="text-align: center;"><i>Comparing and ordering numbers</i></p>	
Sequencing whole numbers, decimal fractions and common fractions and positioning them on a number line	Sequencing decimal fractions, percentages and common fractions, recognising equivalences and translating from one form to another
<p style="text-align: center;"><i>Operations with decimal fractions</i></p>	
Applying place-value concepts to operations involving decimal fractions Applying multiplicative thinking in appropriate situations Applying place-value concepts in meaningful conversion of metric units (eg changing litres to millilitres)	Multiplication and division of decimal fractions
<p style="text-align: center;"><i>Operations with common fractions</i></p>	
Finding fractions of collections Sharing and partitioning	Finding fractions, percentages and decimal fractions of collections Conducting four operations with fractions
<p style="text-align: center;"><i>Working with time</i></p>	
Working out differences in time Using timetables	
<p style="text-align: center;"><i>Written algorithms</i></p>	
<p>Algorithms are routines that are efficient and effective for given purposes. Facility to carry out calculations is an important outcome for students to strive for and to achieve. The ability to select an appropriate method of calculation – mental, written, technology-assisted or a combination of these – is also important. ‘Written algorithms’ in the sense implied here are those pen-and-paper procedures for which number calculations may be performed with whole numbers, fractions and decimals. Algorithms are largely mechanical procedures, but their effective and lasting use depends on students understanding many underpinning concepts such as place value, equivalence and notation, and also the meaning of each operation.</p> <p>To address students’ underperformance, as identified by international and national testing regimes, learning objects should focus on key concepts and on visual and dynamic representations that help students to build appropriate conceptual processes. The classroom teacher could later make use of the understanding that students develop from such learning objects to assist in the learning and application of algorithms where these form part of the curriculum.</p> <p>While the CARG does not require the Project’s learning objects to focus on written algorithms, any learning objects that deal with written algorithms should reflect the following principles:</p>	

- Important underpinning concepts (including place value, alternative and multiple representations) should be addressed in relevant contexts.
- High importance should be given to using and developing appropriate mathematical language. (The use of bytes of audio should be explored.)
- Learning objects should enhance students' understanding of the meaning of operations (including alternative meanings, for example, in partition and quotient in division) and their ability to interpret answers within the context from which the calculation arose.
- Learning objects should focus on common misconceptions; and concepts should be reconstructed from mathematically sound underpinnings.
- Learning objects should make effective use of the medium and technology, rather than mimic 'old' practices.
- Learning objects should highlight and reinforce the links and relationships between the four operations.
- The potential to look at the nature and purpose of algorithms in general should be explored.
- Care should be taken to ensure that any attempt to model and develop concepts and procedures does not become algorithmic itself.
- Written methods should emerge from students' mental computation techniques (see McIntosh and Dole (2004) in '2.8 References for Project Developers' of this PSB.

*Algebraic conventions and procedures*

- Extending observed patterns and sequences
- Expressing generalisations in algebraic form (for example, finding a rule to generate a number pattern) and recognising independent and dependent variables
- Finding values of algebraic expressions by substitution, including recursive expressions
- Simplifying algebraic expressions, including algebraic fractions
- Solution of equations in one variable
- Transposing algebraic formulae and equations
- Solving simultaneous equations by tabular graphical and algebraic methods
- 

Much of the above number-related content relates to students' sense of number, that is a mix of skills, knowledge and understanding to work competently and confidently in contexts in which numbers and quantities are involved.

Developing a sense of number involves:

- growth in familiarity with numbers (whole numbers, fractions and decimals), their meanings and representations
- conceptual understanding of the four basic operations

- competence to perform calculations by mental, estimation, written and calculator methods, including choosing the method appropriate to each calculation and context.

### Applying facts and procedures to solve basic worded problems

Students' underperformance in solving basic worded problems relating to number facts, conventions, procedures and operations is well documented in the TIMSS and PISA reports, as noted earlier. Learning objects directed to improving student performance in the areas listed in the table above should, wherever possible, do so in context rather than in isolation so that students are encouraged to engage with the problem in practical ways.

It is not sufficient to develop learning objects that present a collection of word problems for students to solve. To have a lasting benefit for students, learning objects need to include not only the problems, along with solutions with explanations that students can understand, but also a focus on *how* problems are solved. This requirement, which applies to both routine and non-routine problems, is a challenge to writers and developers that needs to be met.

A 'key words' approach (such as 'of' means 'times') should be avoided because it is unreliable given that word order has such a significant impact on semantic structure. 'Sixty is half of which number?' and 'Half of sixty is which number?' contain the same words but have markedly different semantic structures.

The TIMSS and PISA reports highlight students' underperformance in tasks that require them to select and apply appropriate procedures and strategies to solve one-step and multi-step problems such as those listed below.

Years 3 to 6/7	Years 7 to 8/9
<ul style="list-style-type: none"> <li>• Paraphrasing the statement of the problem and making connections between the problem and the symbols used to represent it mathematically.</li> <li>• Having and being able to apply an effective problem-solving process of:               <ul style="list-style-type: none"> <li>• reading and interpreting the problem</li> <li>• choosing and performing the operation(s)</li> <li>• reflecting on the result of the calculation</li> <li>• relating the outcome of the calculation back to the problem</li> </ul> </li> <li>• Word problems involving basic operations with:               <ul style="list-style-type: none"> <li>• fractions</li> <li>• time</li> <li>• multiplication and division</li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>• Paraphrasing the statement of the problem and making connections between the problem and the symbols used to represent it mathematically.</li> <li>• Having and being able to apply an effective problem-solving process of:               <ul style="list-style-type: none"> <li>• reading and interpreting the problem</li> <li>• choosing and performing the operation(s)</li> <li>• reflecting on the result of the calculation</li> <li>• relating the outcome of the calculation back to the problem.</li> </ul> </li> <li>• Word problems involving basic operations:               <ul style="list-style-type: none"> <li>• involving time</li> <li>• where more than one operation is involved and the order of operations needs to be considered</li> </ul> </li> </ul>

### *Priority: Focus on complex concepts*

In the absence of direction from the Curriculum Forum concerning the intended or implied meaning of 'complex concepts', the Mathematics and Numeracy CARG is adopting a broad interpretation of the term for the purposes of this PSB. 'Complex concepts' is used here to refer to:

- concepts that are acknowledged to be difficult to develop and understand and that therefore may need to be developed over time
- 'mathematically rich' concepts requiring the integration of a number of previously developed and understood concepts such as proportion. (Here, proportion should be interpreted as incorporating ratio, rate and scale and accorded the status of a unifying thread in Mathematics because of its presence in a vast array of contexts and problems that may be solved through proportional reasoning.) There is an opportunity to identify and promote the use of an effective modelling strategy across Years 3 to 9.
- higher-order mathematical concepts such as equivalence and similarity.

Several topics that involve complex concepts but that have yet to be addressed by learning objects for Years 3 to 8/9, including some of those listed below, were identified as areas of minor underperformance in national and international testing. In the case of some other concepts consideration should be given to developing new learning objects to extend the year-level range of their suitability and usage so as to assist students who require more time and other contexts in order to develop their understanding of the concepts. Mapping existing learning objects against 'Statements of learning for Mathematics' (2006) may well identify further complex concepts. In all cases emphasis should be placed on providing students with opportunities to explore complex concepts in meaningful contexts, which model working mathematical processes.

### Topics that employ complex concepts

Students can explore complex concepts for which they can develop strategies and reasoning in topics such as those listed below.

Years 3 to 6/7	Years 7 to 8/9
<ul style="list-style-type: none"><li>• Negative numbers (eg temperatures)</li><li>• Angles and transformations (eg angle sums of polygons, tessellation)</li><li>• Ratio, scale (eg in maps, plans etc) and rates, both formal (eg km/h) and informal (eg lawns cut/hour) in everyday contexts and with attention given to the underpinning proportional reasoning common to them all</li><li>• Inductive thinking via combinatoric (counting) tasks, pattern extension and generalisation</li><li>• Deductive reasoning developed in simple axiomatic environments</li><li>• Interpreting data, particularly using two data sets</li></ul>	<ul style="list-style-type: none"><li>• Operations with integers</li><li>• Interpreting simple quantitative relationships (eg proportional relationships and rates) and performing related calculations</li><li>• Reading whole-number and decimal-number scales with different graduations (as used in measuring instruments)</li><li>• Interpreting map scales and determining distances</li><li>• Understanding latitude and longitude as they apply to basic ideas associated with bearings and direction</li><li>• Proportional reasoning in all its forms (ratio, rates, scale) and in as many different contexts and applications as feasible</li><li>• Ideas associated with proof – deductive and inductive reasoning</li><li>• Understanding the meaning and use of statistical measures such as mean and median</li><li>• Describing and interpreting relationships in text, algebraic and graphical forms.</li></ul>

### *Priority: Problem-solving and open-ended tasks*

Learning objects developed to address this priority should provide for students to develop problem-solving strategies to solve routine and non-routine problems. Contexts need to be presented in which students are required to choose or develop an appropriate strategy to solve a problem. The appropriate strategy might be developing a mathematical model, even one as simple as an equation. To choose such a strategy and use it with confidence, students need to possess some previous experience of the modelling process and to have considered why they might be drawn to using such a strategy. As in real-life contexts, solutions to some problems benefit from multiple pathways and strategies, and some problems have multiple solutions. Linear design of learning objects may not therefore be suitable for developing the level of learner autonomy required by multi-ended or open-ended tasks. Consideration should be given to producing open-ended reusable tools or widgets.

In developing learning objects for problem solving, the importance of problem posing should not be overlooked. In particular, students posing and answering their own questions (as in conjecturing and hypothesising) is a well-used problem-solving strategy in mathematical investigations.

When selecting the mathematical content of the problems, preference should be given to topics listed in *Priority: Focus on complex concepts* or in *Priority: Areas of student underperformance* and *Priority: Epistemic approach to processes, skills and conventions* (see

above). Contexts and settings for problem-solving and open-ended tasks should be engaging, relevant and where possible linked to other mathematical ideas and other learning areas. Teaching *through* problem solving, for which there is considerable potential, should be seen as integral to all mathematics activity and learning and a strong motivator of students' learning. Students should learn problem-solving strategies they can apply in a range of mathematical contexts, and learn mathematical concepts through problem-solving strategies and approaches.

CARG members expressed strong support for the development of resources to assist students in the development of problem-solving strategies and to enhance their performance in open-ended investigations. Sequences of learning objects developed to address this Project priority for the middle years of schooling should be extended into learning objects for the senior years.

### *Priority: Simulations and games*

To meet this priority, learning objects should, within the constraints of time, budget and file size, employ principles of gaming design. Games should incorporate a mix of skill, strategy and chance. A balance must be maintained between mathematical learning outcomes and elements of game design such as engagement, competition, reward, rules, narrative, immersion and enjoyment. The mathematical learning outcomes should not be compromised.

Simulations should allow exploration of mathematical ideas and experimentation with cause and effect factors, and the experience should provide multiple outcomes. Opportunity to make links with problem-solving and open-ended tasks should be sought.

When selecting mathematical content preference should be given to topics listed in *Priority: Focus on complex concepts* or in *Priority: Areas of student underperformance* and *Priority: Epistemic approach- processes, skills and conventions* (see above).

### *Priority: Alignment with 'Statements of learning for Mathematics' (2006)*

- Documentation for the new learning objects produced during Phase 3 will note their alignment with 'Statements of learning for Mathematics' (2006).
- Elaborations to the Statements may be useful for guiding the development of assessment items related to the learning objects.
- A mapping of how all previously developed learning objects align with 'Statements of learning for Mathematics' (2006), yet to be undertaken, may reveal potential areas for the development of learning objects. This mapping will be included in a later version of this PSB.

## **The Senior Years (Years 9 to 12)**

With the exception of a small number of learning objects produced in Projects 1 to 5 that extended to use at Year 9, learning objects for the senior years is a new area of development. Consistent with the original brief set for The Learning Federation, procurement and development of digital resources for the senior years is not intended to cover the entire Mathematics and numeracy curriculum.

The student populations in the senior years of schooling in Australia and New Zealand are increasingly heterogeneous, ranging from students with learning difficulties and those with non-academic aspirations to highly motivated, academically able students. There are associated differences in the mathematical content of the various syllabuses and courses

offered to senior students. The CARG provided the following directions and guidelines for the procurement and development of digital resources for the senior years:

- Procurement and development should not be restricted to any specific population cohort.
- Curriculum mapping of Years 11 and 12 undertaken in Australia (see Masters et al 2006), together with New Zealand curriculum statements, should be used to identify commonalities, especially in the years prior to Year 12 (Australia) and Year 13 (New Zealand).
- The designed time for student engagement and activity with learning objects should be extended to 40 minutes for the senior years, given the longer duration of lessons in these years.
- Procurement and perhaps modification and adaptation of existing digital resources and materials should be investigated, for example:
  - Access to Algebra, MCTP and RIME materials
  - Maths 300 (see Lovitt 2007) materials
  - NCTM (USA) Illuminations materials and the National Library of Virtual Manipulatives
  - Calculus for all approaches, such as in the materials developed in the United States by James Kaput (SimCalc) and in Australia by Mary Barnes.
- Content and contexts should be selected to illustrate and demonstrate the use of statistical and algebraic methods and thinking across the curriculum. The learning objects developed should include simulations and develop students' ability to work mathematically in open-ended and problem-solving environments.
- The opportunity to collect and publish actual data sets for classroom use should be explored.
- Existing learning objects should be reviewed to identify those that could be usefully redeveloped for use in the senior years. In doing so, it is recommended that there be some canvassing of teachers in Australia and New Zealand for opinion. Possible examples include:
  - 'Trigonometry' series: Extending the definitions of trigonometric ratios to angles greater than  $90^\circ$ , including radian measure, and introducing trigonometric functions and their graphs
  - 'The multiplier': Extending the range of numbers and presenting further strategies, including estimation
  - 'Shape maker': Redeveloping it to show 3D objects formed by rotating plane shapes and curves about different axes (solids of revolution) and the shape of cross-sections of 3D objects.
- While acknowledging that financial literacy is the subject of a separate TLF project, financial, business and consumer themes should not be ignored as contexts for learning objects in this Project. An example is the application of recurrence relations (difference equations) in financial contexts such as loan amortisation.

For continuing consistency and coherence with the existing collection of Mathematics and Numeracy learning objects, the content of objects should be identified and selected on the basis of the same priorities as those used for the middle years (Years 3 to 8/9), but applied in the ways described below.

*Priority: Areas of student underperformance*

*Priority: Epistemic approach to processes, skills and conventions*

The areas of underperformance of Australian and New Zealand students identified in the PISA testing program (of 15-year-old students) should guide the selection of content and concepts for developing learning objects for students in the senior years. In the final years of schooling there is increased urgency to address any areas where students have generally underperformed in testing. Even academically able students tend to be underperforming in tests involving contextually based Mathematics needed for everyday life, as their courses of study in Mathematics often focus on higher-order concepts and skills.

## The 2003 PISA testing program

- The PISA program tests mathematical literacy defined as ‘... an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned and reflective citizen’ (OECD).
- Five types of test items are used, ranging from multiple-choice items to openly constructed response items in which students are required to construct a long response that allows for a broad range of divergent, individual responses and differing viewpoints.
- Australian and New Zealand students performed significantly above the average for OECD countries.
- The scope of the testing program is defined by four ‘over-arching ideas’:
  - Space and Shape, relating to spatial and geometric phenomena and relationships, drawing on the curricular area of geometry
  - Change and Relationships, relating most closely to the curriculum area of algebra
  - Quantity, involving both numeric phenomena and quantitative relationships and patterns, and relating to the understanding of relative size, the recognition of numerical patterns, and the use of numbers to represent quantities and quantifiable attributes of real-world objects (counting and measuring)
- Uncertainty, relating to both data and chance, and statistics and probability.
- Analysis of PISA results indicates which of the four ‘overarching ideas’ require additional teaching–learning support for students generally. Of the four ‘overarching ideas’, the only one of relative weakness for Australian and New Zealand students is Quantity, a weakness that appears across the six proficiency levels.
- Analysis of the PISA report suggests that students need teaching–learning support in the following order of priority:
  1. Quantity
  2. Space and shape
  3. Uncertainty
  4. Change and relationships
- Analysis of PISA results reveals not only the content areas that require additional teaching–learning support for students generally, but also the groups of students to whom that support should be particularly strongly directed.

Test performance is reported by PISA at six levels of proficiency (1 to 6), with each student being assigned a level determined by the likelihood (62% probability) of that student responding correctly to any questions at that level.

In the 2003 program 14.3% of Australian and 15% of New Zealand students reached or fell below Level 1 (the lowest) while 5.8% and 6.6% respectively reached Level 6 (the highest). In both countries, Indigenous students were over-represented at the lower levels and under-represented at the higher levels.

About one-third of students (32.9% in Australia and 34.2% in New Zealand) are performing at Level 2 or below it, some of them even below Level 1. PISA describes performance at these lowest two proficiency levels in the following terms:

### *Level 2*

- Students can interpret and recognise situations in contexts that require no more than direct inference.
- They can extract relevant information from a single source and make use of a single representational mode.
- Students at this level can employ basic algorithms, formulae, procedures, or conventions. They are capable of direct reasoning and making literal interpretations of the results.

### *Level 1*

- Students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined.
  - They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations.
  - They can perform actions that are obvious and follow immediately from the given stimuli.
- 
- While there are significant gender differences in the results for the OECD overall (in favour of male students), Australia was one of seven OECD countries in which there was no statistically significant gender difference in the test results. However, there is evidence of sufficient gender difference in the Australian and New Zealand test results to indicate the need for care in selecting contexts for all new learning objects.
  - Language ('English spoken at home/not spoken at home') is a significant factor affecting test performance, particularly for the overarching idea of Uncertainty, the area in which the test performance of Australian students showed the widest differences.

### *Priority: Focus on complex concepts*

In the absence of direction from the Curriculum Forum concerning the intended or implied meaning of 'complex concepts', the Mathematics and Numeracy CARG is adopting a broad interpretation of the term for the purposes of this PSB. 'Complex concepts' is used here to refer to:

- concepts that are acknowledged to be difficult to develop and understand and that therefore may need to be developed over time
- 'mathematically rich' concepts requiring the integration of a number of previously developed and understood concepts such as instantaneous speed and acceleration
- higher-order mathematical concepts such as invariance and limit.

The selection of 'complex concepts' as a priority for the development of learning objects in this Project has been informed by students' results in the PISA testing. As part of the general practice of cyclically revisiting and extending many mathematical concepts over the school years, in the senior years there is a need to develop increasingly broader and deeper understanding of such concepts, embedding their applications in contexts that are relevant and meaningful to senior students.

There is potential to extend the development and application of concepts that appear in the collection of learning objects developed during Phase 1 and Phase 2 of the initiative. To this end writers and developers of learning objects for the senior years should take account of the following suggestions.

- 'The mathematics of change' should not be interpreted solely as a study of differential calculus. Most situations in which mathematical modelling approaches are used involve finding specific values of variables and/or quantifying and interpreting change in variables. This might be as simple as solving an equation or finding and interpreting

the slope of a linear function, or as complex as interpreting change in exponential function models of growth and decay. Learning objects focusing on change and rate of change should draw on contexts in the sciences, finance and other areas of the curriculum.

- At the higher levels of Mathematics courses offered in most if not all jurisdictions, complex concepts suited to development of learning objects include those associated with differential calculus. For example, a learning object (simulation) that graphically and numerically deals with the construction and evaluation of the left and right derivative for a given point at  $x = a$  by secant construction and informal approach to limiting value.
- An example for integral calculus may include a learning object (simulation) that graphically and numerically deals with the construction and evaluation of the sum of left and right rectangles for a given interval  $(a, b)$  and informal approach to the corresponding limiting value.
- A series of learning objects to introduce the notion and use of scatter plots to investigate relationships between continuous variables was developed in Project 5. For the senior years there is an opportunity to extend this theme to a learning object that enables students to interactively explore the ideas underpinning least squares regression – a graphical and numerical approach to carrying out least squares minimisation on a data set. For a data set whose points are plotted (students could input these points or some of them, or initially be given a set), students could adjust (by eye) the gradient and/or intercept of a straight line of good fit while the corresponding sum of squares of differences (predicted or actual) for the data set is interactively calculated. When students have obtained what they think is a line that appears to minimise this value, their line could be compared with the actual least squares regression line of best fit. In related parts of the activity students could consider the use of this line for predictive purposes; the effect of varying one or more points (ie more like outliers) on the equation of the line of best fit; and the coefficient of determination as a measure of the amount of variation in the dependent variable attributable to variation in the independent variable.
- For differential equations, a learning object could give a direction (gradient or slope) field for a specified differential equation.
- In the senior years proportional reasoning is a unifying theme for algebra, geometry, measurement and chance and data. Being able to think and reason using multiplicative techniques is an important problem-solving strategy. As with the development of other problem-solving strategies, it is important to have a variety of problem settings and also a variety in the opportunities provided for reflecting on how and why such strategies are useful and why they are chosen.
- Beyond Year 9, in all jurisdictions and for many students, the development of algebraic concepts and skills (for example, quadratic functions and equations, equation solving) takes on increasing prominence. Opportunities for the simultaneous development and application of these skills and concepts in relevant contexts should be sought. Algebraic modelling of real-world situations should feature in the learning objects.

These examples do not 'fit' exclusively under *Priority: Focus on complex concepts*, nor are they unrelated to one another. Clearly some would involve the construction of simulations and tools and take advantage of the power, flexibility and interactivity that the digital medium provides. While it is true that some of these ideas can be developed through the use of graphics calculators, learning objects offer opportunity for greater interactivity and scaffolded feedback.

### *Priority: Problem-solving and open-ended tasks*

Learning objects developed to address this priority should assist and encourage students to develop and use problem-solving strategies and techniques and highlight the benefits of such learning and actions. Opportunities to develop skills and concepts (identified and described elsewhere in this scoping brief) should occur wherever possible in problem-solving, investigative and mathematical modelling contexts, in contrast to ‘first the skills, now some problems’ approaches.

It is important that students be provided with challenging problem-solving situations in which they are required to choose a suitable strategy from their personal repertoire. This presents a challenge for the developers of learning objects. The file-size limit for a learning object (now 1 megabyte), the time limit for engagement in an activity (extended to 40 minutes for the senior years) and customary scaffolding and feedback are hallmarks of the learning objects developed during Phase 1 and Phase 2. However, it will be difficult to maintain these characteristics in the development of learning objects featuring open-ended problems that permit multiple pathways and strategies for their solution and that may even have multiple solutions that all satisfy the demands of the task. The heavily scaffolded linear design of the learning objects developed in Phase 1 and Phase 2 may not be suited to encouraging the greater learner autonomy required by multi-ended or open-ended tasks.

CARG members expressed strong support for the development of resources to assist students in the development of problem-solving strategies and to enhance their performance in open-ended investigations. Learning objects developed to address this priority in the senior years should build on the problem-solving strategies being introduced and developed for the Project’s middle years. Students need experiences in which they apply previously learnt strategies to new and more sophisticated problems and contexts as well as opportunities to broaden their problem-solving techniques through exposure to and use of new strategies.

### *Priority: Simulations and games*

Students in today’s senior year’s classes have had lifelong access to computer-based and hand-held electronic games (whether for education or entertainment) that have been developed with few constraints and with budgets that are based on worldwide distribution and sales. Learning objects, within the constraints of time, budget and file size, should employ principles of gaming design. They should incorporate a mix of skill, strategy and chance and feature elements of game design such as engagement, competition, reward, rules, narrative, immersion and enjoyment, but without compromising mathematical learning outcomes.

Simulations should allow exploration of mathematical ideas and experimentation with cause and effect factors, and the experience should provide multiple outcomes. Simulations should make explicit any underpinning assumptions that differentiate between the simulated world and the real world. Students should be assisted to interpret and translate the outcomes of the simulations to real-world contexts and make judgements about the relevance, significance and implications of the results. Contexts should be selected to highlight the usefulness of simulation and mathematical modelling in settings that are not overtly mathematical and that require the integration of various mathematical concepts and strategies.

Some tools (simulations and mathematical models and representations) featured in the learning objects developed during Phase 1 and Phase 2 of the initiative could be extracted from their heavily scaffolded environments, adapted, and presented as tools for open-ended investigations and explorations. Examples include:

- the ‘Random distribution’ series, which offers an opportunity to vary the packet size and compare empirical data with theoretical (calculated) expectations.

- the 'Shape maker' series, which offers an opportunity to extract the rotation tool and increase the range and description of shapes able to be rotated, so as to generate solids of revolution for use in integral calculus contexts.
- the 'Trigonometry' series, which offers an opportunity to extract the unit circle tool and extend the angle range beyond 90°, using the tool to generate graphs of trigonometrical functions and to explore simple harmonic motion.

*Priority: Alignment with 'Statements of learning for Mathematics' (2006)*

While the scope of 'Statements of learning for Mathematics' (2006) extends only to Year 9, the Statements will heavily influence the choice of content and alignment of learning objects to be developed for students in the senior years as the Statements establish the platform for programs of learning in the senior years. Because the Statements provide the foundation on which subsequent programs of learning will be built, learning objects developed within the Project need to be aligned to the Statements.

Learning objects developed for students at Years 10 to 12 need to link with and build on the knowledge, understandings and skills described in 'Statements of learning for Mathematics' (2006).

- Documentation for the learning objects produced in Phase 3 will note their alignment with 'Statements of learning for Mathematics' (2006).
- Elaborations to the Statements may be useful for guiding the development of assessment items related to the learning objects.
- A mapping of how all previously developed learning objects align with 'Statements of learning for Mathematics' (2006), yet to be undertaken, may reveal potential areas for the development of learning objects for the senior years. This mapping will be included in a later version of this PSB.

## 2.5 Teaching and Learning Issues

While the scope of this PSB has been informed and influenced by reports of student underperformance in international and national testing programs, it is important that the future procurement and development of learning objects continues to be based on developing and maintaining the coherence of the collection developed in Projects 1 to 5. 'Curriculum patching' or a 'quick fix' approach to improving student performance is discouraged. The new learning objects are not intended to provide solutions to all problems.

The following characteristics of the existing collection provide a guide to the procurement and development of new learning objects.

- Learning objects enable students, individually and collectively, to work with complex content and ideas in new and dynamic ways.
- Learning objects provide for forms of multimodal learning that, due to its complexity, along with its safety, time and cost requirements, is not usually able to occur in normal classrooms.
- Learning objects challenge students to question, investigate, analyse, synthesise, solve problems, make decisions and reflect on their learning.
- Learning objects contain scaffolded learning tasks, providing students with feedback on their learning in a variety of supportive and engaging ways.
- Learning objects are used by teachers in a range of ways to meet the needs of their curriculum and the various needs of their students.

As with Projects 1 to 5 for Years P to 9, there is no intention that the new learning objects should deal with all parts of the content range of the Mathematics curriculum, nor that they should replace the teacher in the classroom.

## 2.6 Pedagogy and Instructional Design

In general, the pedagogical approach and resultant instructional design must:

- be based on the view that people construct new knowledge and understandings based on what they already know and believe
- use the above constructivist approach to progress students' overall grasp of important mathematical content and its coherence
- provide rich, meaningful contexts demonstrably capable of engaging and sustaining student involvement
- assume that students have had previous concrete experiences and will continue to have further ones
- avoid linear approaches to learning in which ideas are presented step-by-step in atomised pieces; and instead deal, at least initially, with substantial mathematical ideas
- take advantage of situations in which knowledge, skills and understandings may be developed in problem-solving contexts
- be inclusive of all students.

In the middle years (Years 3 to 8/9) domain of this project, a priority in the selection of content and context should go to those areas of the Mathematics and Numeracy curriculum in which students have underperformed in international and national assessment programs and in which learning objects are considered to be relevant and useful in the teaching and learning process. It should be accepted that some of the areas of underperformance may be best dealt with by teachers using other resources.

In the senior years (Years 9 to 12) domain of this project the brief is less clearly defined. Content areas from the middle years need to be extended and further developed. Contexts in which some ideas and concepts have been previously introduced but not fully or successfully developed need to be revisited in more age-appropriate and sophisticated contexts in the senior years. Development of digital resources for the senior years requires identification of difficult-to-teach concepts for which learning objects could provide assistance, motivation, engagement and novel contexts.

In the development of the new digital resources, experience in Projects 1 to 5 suggests that opportunities should be sought for greater and earlier involvement of practising classroom teachers in the development cycle. Such earlier involvement of teachers may provide software developers with greater understanding of the potential role of the learning objects within particular teaching and learning sequences, thereby ensuring a good fit with commonly undertaken courses of study and meeting the goal of minimising 'stand-alone' use of learning objects.

## 2.7 Learners' Characteristics

Online content for The Le@rning Federation is developed using a user-centred methodology. The following profiles of the student users should assist instructional designers and multimedia developers to understand the characteristics of the students for whom they are working. It needs to be understood that while learners in these year groupings typically display the general characteristics set out below, all learners are different and no one is typical. Students' conceptual understandings, preferred learning modes and skill development vary widely. There is often variation between communities and within communities.

As there are marked variations in the characteristics of students within the Project's two broad domains (the middle years and the senior years), the characteristics of the students who will use the new learning objects are presented below in five groups that overlap to form a continuum.

## Years 3 and 4

Although interested in their personal world, these students' sense of themselves and their world is expanding. They are beginning to see themselves as members of larger communities and as part of their local environments. They are beginning to enjoy investigating and finding out about some environments and phenomena in the wider world, particularly when they are able to see relevance and make a personal or everyday link to the subject of study.

Play, interaction and communication continue to be important sources of ideas, stimulating these students' creativity and imagination. They connect real and imagined experiences from the past, present and future and are beginning to make sense of these. They enjoy expressing themselves creatively and although becoming more self-conscious, are generally still willing to take risks. Their hand-eye coordination is usually improving, as are their fine motor skills and their organisational and cooperative skills. Many are beginning to understand and appreciate different points of view and are developing a respect for others. They can generally distinguish between beliefs and rules. They can recognise the cause and effect of changes to real, concrete events and objects. Some students show early signs of alienation from schooling and can exhibit challenging behaviours in a variety of educational and social settings.

Students in these years continue to be visually and physically oriented. It should, however, be noted that some have significant disabilities that make these orientations a challenge.

These students generally have developed a longer concentration span that enables them to sustain activities for longer periods. They can also think in more abstract terms than in Years P to 2 and are able to interrelate numeracy ideas from various sources. Their ability to infer is emerging but these inferences and assertions may be quite simplistic.

National testing data shows that 92.7% of Australian students reached the national numeracy Year 3 benchmark in 2001 and 93.9% in 2000. The national benchmark represents the minimum acceptable standard of numeracy without which a student will have difficulty making sufficient progress at school.

The *National Report on Schooling in Australia, Preliminary Paper 2000* states that the following examples are typical of the skills required to meet the Year 3 numeracy benchmark:

- Read and write whole numbers up to 999
- Demonstrate their knowledge of place value (eg know that 86 can be written as 8 tens and 6 ones)
- Remember, or work out, basic addition facts to  $10+10$ , the matching subtraction facts (eg  $9 + 4 = 13$ ,  $13 - 9 = 4$ ) and extensions of those facts (eg  $23 - 9 = 14$ )
- Add and subtract whole numbers to 99, by using mental and written methods or by using a calculator
- Solve simple problems set in familiar situations (eg work out how many of the 26 children are left in the classroom if 12 go to the library)
- Add up coins (up to \$5.00) and know whether they have enough to buy a particular item [Note: Since October 2006, 5 cent coins are no longer in circulation in New Zealand.]
- Tell the time in hours and minutes on digital clocks and hours and half-hours on analogue clocks
- Collect and organise information, display it in simple bar graphs or picture graphs, and comment on the information
- Recognise and name familiar 2D and 3D shapes and objects (ie triangle, square, rectangle, circle, cube and pyramid)
- Use language that shows they understood position and direction when using a simple grid, map or plan (eg the tree is between the house and the fence.)

Provided they have access to effective, well-networked and supported information and computer technologies at school and at home, their computer skills are becoming increasingly well developed.

## Years 5 and 6

In the ACT, New South Wales, Victoria and Tasmania, Year 6 is the last year of primary school. Some middle schools span Years 5 to 9. In other Australian States Year 7 is the concluding year of primary school while in New Zealand it is Year 8.

In all of these schooling arrangements, students at Years 5 and 6 are demonstrating stronger personal initiative and some are becoming more assertive. They are increasingly identifying as members of groups such as peer groups or teams and are becoming more interested in peer-related concepts and the popular media. This is typically reflected in their enjoyment of learning from and with others. Their organisational skills are improving further and they are often able to work independently as well as cooperatively. They are often imaginative and enthusiastic and enjoy problem solving.

While some students exercise leadership responsibilities at this level of the primary school others are already alienated, sometimes exhibiting lack of interest in schooling, underperformance and challenging behaviours. Most students can articulate their own beliefs and recognise different beliefs in others.

At this level, student's abstract thinking capacities are developing. They are now able to deal with more than one concept at a time but most still find it difficult to comprehend abstract concepts. They still tend to work at a concrete level. From their observations they are able to see patterns and make generalisations, for example, they can extend their generalisations to make predictions of events.

Most are literate in English and/or their first language, as shown by 87% of Year 5 students having reached the national reading benchmark in 2000. It should be noted, however, that achievement of the benchmark varies widely among student populations. Performance of particular student populations can vary from 98% to 34% depending on their geographic location and language background.

In 2000 and in 2001, 89.6% of Australian Year 5 students reached the national numeracy benchmark, which represents the minimum acceptable standard of numeracy without which a student will have difficulty making sufficient progress at school.

The *National Report on Schooling in Australia, Preliminary Paper* (2000) states that the following are typical examples of the skills required to meet the Year 5 numeracy benchmark:

- Read, write and use whole numbers up to 9999 and place them in order of size
- Show an understanding of simple fractions (eg work out one-third of the students in the group to form one of three equal teams)
- Show understanding and use decimals in familiar contexts (eg say that a rope 3.7 metres long is longer than a 3.65 metre rope; explain that a 1.25 litre bottle holds more than a 1 litre bottle, but less than a 2 litre bottle)
- Work out the answers to addition and subtraction problems that involve three-digit whole numbers or money, and decide the most appropriate way to do that (ie mentally, by written methods or using a calculator)
- Perform simple multiplications and divisions with whole numbers such as  $34 \times 6$ ,  $36 \div 3$ , by using mental or written methods
- Make mental and written calculations involving money (eg work out mentally the change from \$10.00 for an item worth \$2.90)
- Interpret measures expressed in decimal form (eg know that a measure of 1.5 litres is one and a half litres; know that a jump of 2.95 metres is nearly three metres)

- Tell the time in hours and minutes on analogue and digital clocks
- Describe and compare 2D and 3D shapes and objects according to their important features (eg say why a shape would be a cone rather than a cylinder)
- Use conventional terms such as angle, face edge and base to name parts of 2D and 3D shapes.

Provided they have access to computer technology at school and at home, many students at Years 5 and 6 are computer-literate and feel increasingly technologically capable.

Students at this level are generally physically active and energetic, although physical disabilities and (increasing) levels of obesity pose challenges. At this age, some students have already reached puberty.

## **Years 7 and 8**

Students at Years 7 and 8 align strongly with their peer groups. Their choice of peer group reflects shared interests, values and beliefs. Often self-conscious, their self-image is greatly influenced by their level of acceptance within their peer group; and their choices and preferences increasingly reflect those of their peers and the influences of the media and popular culture. While for some students the influence of their families or caregivers may be declining, for others it remains very strong. Most still accept their family's beliefs and values without question but some begin to reflect upon the differences between their beliefs and values and those of others.

Although they are engaging with broader perspectives both in the community and at school, their interests extend well beyond their own communities and they are developing concerns about wider issues. Typically they are interested in the natural, social and technological world and its impact on their current and future lives. Some may begin to question established conventions, practices, values and adult priorities. Some may be developing an interest in particular fields for the personal satisfaction it provides.

Their thinking is becoming increasingly abstract. They like a challenge. They can recognise significant issues, pose focused questions and plan and carry out investigations. Their ability to distinguish between fact and opinion is improving. They can justify generalisations, identify cause and effect and use evidence to support an argument or point of view. Most can think logically and analytically and may use mathematical concepts and generalisations to suggest explanations for phenomena rather than looking for specific facts. They are beginning to be able to give explanations for their concrete, evidence-based generalisations using ideas that do not rely only on their senses. They recognise that explanations for single events and phenomena can be made using a simple concrete model.

While many students at Years 7 and 8 are taking more responsibility for their own learning, some may be alienated from schooling. A sense of alienation reduces their learning commitment and levels of knowledge and skill, and contributes to a sense of failure in both male and female students, all of which is usually more evident in male students.

Generally students at this level are literate in English, which is strongly correlated with success in secondary school. Students who are succeeding in literacy are much less likely to feel a sense of alienation and failure.

Provided students have access to computer technology at school and at home, they are relatively confident with this technology and feel they have both breadth and depth of skills. As a consequence, they are looking for innovative and targeted use of ICTs in their learning programs, having mastered the basics at a relatively early age. Learning activities, therefore, need to build on and broaden students' ICT skills base using innovative and challenging approaches that explore concepts and issues in a rich and deep manner, using both actual and virtual experiences to enrich that learning.

Year 7 students are expected to be able to use whole numbers to seven digits and numbers with decimal fractions to two decimal places in familiar situations (eg money and measurements). They are expected to be able to record these in different formats, using place value knowledge. They are also expected to recognise and use common fractions and equivalences between fractions, decimals and key percentages (ie 10%, 20%, 25% 50%, 100%). They should be able to use numbers to create and continue patterns. They should also be able to add, subtract, multiply and divide using a variety of strategies and to apply these skills to solve problems in contexts familiar to them.

In relation to measurement and data sense, students should be able to use common measuring instruments to measure and compare length, capacity, mass and temperature, and estimate using a range of standard units. They should be able to measure area and volume by counting units. They should recognise large and small standard units.

As far as uncertainty and predictability are concerned, discussion of variation within chance settings becomes important, and students should be introduced to more formal statistical skills, such as calculation of means and more complex data displays in the form of tables and graphs.

These students can order and quantify the probabilities of outcomes on the basis of trialling. They recognise that variation occurs between different samples of the same event, and are also able to attribute approximate proportions to the outcomes. With deterministic events such as dice games, they can use systematic approaches (such as drawing tables and diagrams) to find all possible outcomes. From these theoretical models they can predict the probability of outcomes in trialling, recognising variation in sampling.

Students can identify issues and topics of particular interest and create, trial and refine questions that allow for appropriate details to be gathered through surveys, interviews and existing sources. They can organise data and experiment with a variety of manual and electronic displays and select those that represent the data clearly. They can make statements regarding the results of their surveys using quantitative and comparative language

Many students by this age have developed a certain cynicism about 'uncertainty' and 'predictability' that at times clouds their understandings and appreciation. The interactive, questioning aspects of learning objects have the capacity to broaden their understandings in a way that conventional methods are unable to do.

## **Years 9 and 10**

Whereas, outwardly at least, a large proportion of students continue to develop along lines established in Years 7 and 8, the Years 9 and 10 cohort of students is becoming increasingly diverse in a number of ways.

These students continue to align themselves strongly with their peer groups, but peer groups are now characterised by increased variety and more specialised shared interests, values and beliefs. In becoming much more aware of and influenced by the media and popular culture, many students are becoming more self-conscious, more self-aware and more dependent on being accepted by their peer groups. They may look to acceptance of more than one peer group, perhaps because new class groupings arising from course choices begin to influence friendships and associations.

For some Years 9 and 10 students the influence and support of their families or care-givers may be strong and unaltered from that experienced in Years 7 and 8, while for others such influence and support may be continuing to decline.

If individually or collectively interested, these students can respond to a challenge, becoming self-directed and creative learners, demonstrating initiative and problem-solving capacity, able to plan and manage complex tasks, both individually and collaboratively. Some are becoming

aware of the tentative nature of their generalisations and are leaving those generalisations open to logical consideration of alternatives.

Through their recognition of and engagement with broader perspectives and issues such as equality of opportunities, gender, community values, justice and beliefs, students' interests continue to extend beyond their own school and community to wider, even global, contexts. Difference and diversity in the cohort is sometimes distinguished by emerging patterns in the ways students are coming to think and reason, even though individual students themselves are sometimes unaware of this development within themselves. These emerging patterns are often influenced by students' growing scholastic strengths and sometimes strongly influenced and nurtured by positive connections with teachers. For some, their school experiences and achievements in areas of their strength and interest are beginning to influence their career aspirations.

As young people in Years 9 and 10 become more physically and emotionally mature, so too are they becoming increasingly complex thinkers, generally able to structure, sequence and link ideas. Some develop their own preferred learning style. Most have a stronger sense of their own strengths, interests and goals. They may be focused on their future and beginning to consider and experience vocational opportunities and they expect to play a major role in determining the decisions that affect their lives.

Their sense of themselves as active participants who have roles to play and responsibilities to exercise in the wider world is developing. They are often concerned about major environmental and social issues, including ethical implications, and are developing an awareness of the complexity of their interdependence. Many have definite views although sometimes they overstate or fail to justify their opinions. They have a heightened sense of justice but may have difficulty reflecting on their beliefs and the bases for those beliefs. They have an increasingly sophisticated understanding and use of subtlety, irony and humour. They often take part in discussions about personal and social issues, especially in relation to rights and responsibilities. While these students do not always act in socially responsible ways, they may have a sound understanding of the issues at stake.

Most students have well-developed literacy skills. Those who are succeeding in literacy are much less likely to feel a sense of alienation and failure and to be at risk of not completing Year 10. Some students may lack motivation and engagement with tasks and even be actively disruptive or passively withdrawn. While this happens with both male and female students, but the impact on young men is more evident in these years.

As at Years 7 and 8, provided students have access to ICTs at school and at home, they are confident with this technology and feel they have both breadth and depth of skills.

The extent to which students need to master specific mathematical content, as they do in the earlier years, is a matter of debate. This cohort of students is becoming increasingly diverse in their individual mathematical ability, the mathematical content of their courses and in their levels of engagement, achievement, interest and motivation in relation to Mathematics.

At this stage of schooling, different series of learning objects might be developed for different subpopulations of students. What is important across the board, however, is that the contexts chosen for learning objects should be embedded in 'real-world' settings that are recognisably relevant and meaningful to the age group and that the on-screen presentation of the learning objects should suit the context and the age group. Even when the mathematical concepts involved may be basic, students at this level will not tolerate childish contexts in texts and graphics.

## **Years 11 and 12**

There is significant variation in how the various States and Territories and government and non-government sectors provide education for their Years 11 and 12 students. In some schools Years 11 and 12 classes form part of a single-site Years 7–12 secondary school,

while in others there is a separate Years 11–12 senior secondary campus. In some jurisdictions TAFE and evening classes offer similar Years 11–12 courses to those offered in school settings.

The cohort of Years 11 and 12 students is diverse in its age composition, educational needs and career aspirations. The ages of students in Years 11 and 12 varies from 15 years to mature age. In regular school settings most students are aged 15 to 19 years, but in senior secondary colleges and settings open to mature age re-entry, a wide range of ages is not uncommon. In the lack of uniformity in its age composition, the Year 11 and 12 cohort differs from the younger cohorts.

Associated with this wide age range are varying degrees of student independence and varying lifestyles. Even among the 15-to-19-year-olds, while most are still living with their families or care-givers, some students have had to move to boarding accommodation in bigger centres in order to access Years 11 and 12 courses, others have to mix work with study in order to support themselves independently, and some students are living in relationships or with peers. Associated with their degree of independence, their responsibilities and their living arrangements there are noticeable differences in students' engagement with and commitment to learning. These differences manifest themselves in students' ambition, determination, concentration, motivation, resilience, engagement and ability to deal with pressure, whether it is self-imposed or external.

Despite these differences, students in the senior years possess some characteristics arising from their age, maturity and previous school experience that distinguish them from the younger cohorts.

- Many students in the senior years have developed strategies, albeit to varying degrees, to learn material that needs to be recalled or used at a later date. They have 'worked out' for themselves what deliberate actions they have to take, and what deliberate actions are effective and successful for them.
- Many students have made progress in learning how they learn: that is, what learning styles they prefer and that succeed for them. Many are beginning to reflect on this aspect of their learning and to perceive how it could shape their futures.
- Most students have superior skills of logical (or at least qualified) argument, reasoning and analysis than do their younger counterparts. Some of this comes from their more mature years and wider experience. The more academically able groups are becoming distinguished by their ability to reflect on, differentiate between and use various reasoning styles, for example, deductive, analytical, inductive and quantitative.
- Many students have acquired problem-solving techniques and strategies through successes with previous problem-solving tasks. They are prepared to consciously apply these to new and unseen problems and are confident in their ability to transfer knowledge, skills and strategies to novel situations.
- Most students in these senior years are quick to judge the content of their learning on the basis of being able to see a connection between this learning and their own personal, not-too-distant futures.

## 2.8 References

Previous Project Scoping Briefs have included extensive lists of references to support each project's theme. However, the entries below relate to only some of the new Project's themes emerging from the Project's objectives and its concept and content priorities. The list below will be expanded as the scope of the project is refined and the results included in later versions of this PSB.

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